Middle School Mathematics Manual

By

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Dedication

To African People,
the race of men and women who enlightened and educated the world not only in mathematics, science, and the arts, but in civility, bravery and superior spirituality.

To All of the Africans at home and abroad.

To the African Ancestors,
who by their great deeds of yesterday continue to inspire us today.

As Black people were the first architects, artists, engineers, mathematicians, physicians, pyramid builders and scientists, this manual is but a modest and humble attempt at the continuation of the great skill and talents that Black people have been endowed with since time immemorial.

LORD WHY DID YOU MAKE ME BLACK [part 1]

Author Unknown

Lord, Lord,
Why did You make me Black?
Why did You make me someone
The world wants to hold back?

Black is the color of dirty clothes;
The color of grimy hands and feet.
Black is the color of darkness;
The color of tire-beaten streets.

Why did you give me thick lips,
A broad nose and kinky hair?
Why did You make me someone
Who receives the hatred stare?

Black is the color of a bruised eye
When somebody gets hurt.
Black is the color of darkness.
Black is the color of dirt.

How come my eyes are brown and not the color of the day-light sky?
Why do people think I'm useless?
How come I feel so used?
Why do some people see my skin and think I should be abused?

Lord, I just don't understand;
What is it about my skin?
Why do some people want to hate me
And not know the person within?

Black is what people are "listed",
When others want to keep them away.
Black is the color of shadows cast.
Black is the end of the day.

Lord, You know, my own people mistreat me;
And I know this isn't right.
They don't like my hair or the way I look
They say I'm too dark or too darn light.
Lord, Don't You think it's time
For You to make a change?
Why don't You re-do creation
And make everyone the same?

Source: Internet

LORD WHY DID YOU MAKE ME BLACK? [part 2]

Author Unknown

God answered saying:

Why did I make you Black,
You have the audacity to ask?
I did not do it as a joke,
Or as some cruel task.

Get off your knees and look around.
Tell Me, what do you see?
I didn't make you in the image of darkness.
I made you in the Likeness of Me!

I made you the color of coal
From which beautiful diamonds are formed.
I made you the color of oil,
The Black gold that keeps people warm.
I made you from the rich, dark earth
That grows the food you need.
Your color is the same as the panther's
Known for her beauty and speed.

Your color's the same as the Black stallion,
A majestic animal is he.
No! I didn't make you in the image of darkness
I made you in the Likeness of Me!

Your hair is the texture of lamb's wool;
Such a humble, little creature is he.
I am the Shepherd who watches them.
I am the One who will watch over thee.

You are the color of dark clouds
Formed during my strongest winters in December.
I made your lips full so
When you kiss the one you love...they'll remember.

Your stare is strong; your bone structure,
Thick....to withstand the burdens of time.
The reflection you see in the mirror...
The Image looking back at you is MINE!

So in answer to all of your questions,
And to forgive you for all of your flack;
These are the reasons, I THE LORD
Made you SO BEAUTIFUL AND BLACK.

Source: Internet

Table of Contents

Definitions: A to L

a.m.	a way of expressing time between 12 midnight and 12 noon
acute angle	an angle measuring less than 90 degrees
addends	numbers to be added together in an addition problem
angle	the measured space between two lines that meet at one point
area	the number of square units needed to cover a surface
arithmetic	having to do with addition or subtraction of terms
average	a number obtained by adding a group of numbers together and dividing by the number of addends; it is the same as the **mean**
center	the point from which all points on a circle are equally distant
circle	a closed, curved line. Each point on the circle is the same distance from the center of the circle
circumference	the distance around a circle
congruent	being the same
decade	a period of 10 years
degree	a unit used to measure an angle or temperature
diameter	a line segment across a circle that contains 2 points on the circle and passes through the center point
difference	the answer to a subtraction problem
→ divisor	a number by which another number is to be divided
→ dividend	the number being divided by the divisor
→ quotient	the answer to a division problem

Example: ***10 ÷ 5 = 2*** ; 10 is the dividend, 5 is the divisor, 2 is the quotient

exponent	a number that shows how many times another number is to be used as a factor; $8^3 = 8 \times 8 \times 8 = 512$, where 8 = base; and 3 = exponent
geometric	having to do with a multiplication or division of terms
integer	a whole number either positive, negative or zero
interest	a fee paid for the privilege of borrowing money
intersecting	passing through or cutting across
inverse operations	the opposite operation
Example:	Multiplication and division are inverse operations. You can use multiplication to undo division. You can use division to undo multiplication.

least common denominator [LCD]

the lowest common multiple of two or more denominators

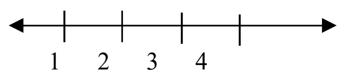
least common multiple [LCM]

the lowest number (not a zero) that is a multiple of two given numbers

Definitions: L to P

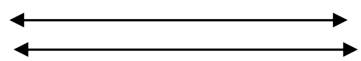
line	a straight path that has no end in either direction
line segment	part of a line - it has a beginning and an end
mean	a number obtained by adding a group of numbers together and dividing by the number of addends; it is the same as **the average**
median	the middle number in a set of numbers arranged in order; when there isn't a middle number, the median is the average of the two middle numbers
metrics	unit of measurement: centimeters, kilograms, millimeters
negative number	a number that is less than zero
number line	a line that shows where numbers fall in order

Example:



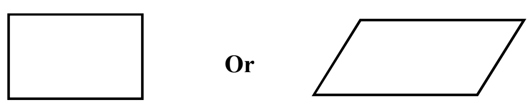
obtuse angle	an angle measuring more than 90 degrees
of	a way of expressing multiplication
p.m.	a way of expressing time between 12:00 noon to 12 midnight
parallel lines	two lines, the same distance apart that never intersect

Example:



parallelogram — a quadrilateral with two pair of parallel sides

Example:



parentheses	( ) means multiplication
percent	a way to compare a number to 100

Example: $25 is 25% (percent) of $100 because:

$$\text{Percent} = \frac{\text{part}}{\text{whole}} = \frac{\text{part}}{\text{total}} = \frac{25}{100} = 25\%$$

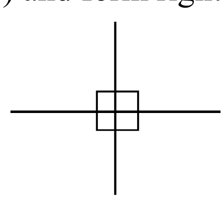
Note: Percent and Probability are one in the same

perimeter — the distance around a figure; the sum of all of its sides

perpendicular lines

two lines that intersect (cross) and form right angles (90° angles)

Example:



Definitions: P to R

<u>place value</u> — the value given to the place a digit occupies in a number 427 (4 is in the hundreds place, 2 is in the tens place and 7 is in the ones place)

<u>polygon</u> — a figure bounded by three or more sides

<u>positive number</u> — a number that is more than zero

<u>profit</u> — money received from a business venture after all expenses have been paid

<u>predict</u> — to guess what will happen

<u>prime number</u> — a number that can only be divided evenly by itself and 1

<u>probability</u> — the likeliness that something will happen; mathematically, $\text{Probability} = \frac{\text{part}}{\text{whole}} = \frac{\text{part}}{\text{total}}$

Note: Probability and Percent are one in the same

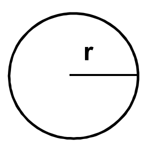
<u>product</u> — the answer to a multiplication problem

<u>quadrilateral</u> — a polygon with four sides

<u>quotient</u> — the answer to a division problem (other than a remainder)

<u>radius</u> — a segment from the center of a circle to a point on that circle

Example:



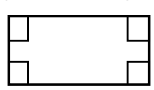
radius = r

<u>ratio</u> — two numbers used to make a comparison

Example: 2 to 1 is written 2:1

<u>rectangle</u> — a figure that has two sets of parallel sides and four right angles

Example:



<u>remainder</u> — the number that is left over when a number cannot be divided evenly

Example: if you divide 7 by 3, the quotient is 2 with a "remainder of 1"

<u>right angle</u> — an angle measuring 90 degrees

<u>right triangle</u> — a triangle that has one right angle

<u>rounding</u> — replacing a number with another in units of tens; numbers 1 to 4 are rounded down while 5 to 9 are rounded up

Example: 32 rounded to the nearest ten is 30; 37 rounded to nearest ten is 40

Definitions: S to W

<ins>sale price</ins> a price that is lower than the original price for an item
<ins>sales tax</ins> an amount of money added to the price of an item that is paid to the government
<ins>sequence</ins> a series of numbers coming one after another in a certain order
<ins>set</ins> a group of items
<ins>square</ins> a figure with 4 right angles and 4 equal sides
<ins>square root</ins> $\sqrt{}$; a number that when multiplied by itself will produce a certain number

Example: $\sqrt{169}$ = the square root of 169 = 13;

<ins>standard form</ins> the way in which numbers are usually written

Example: 6,852

<ins>sum</ins> the answer to an addition problem
<ins>tip</ins> an mount of money paid to a person such as a waitress or waiter to say than you for good service, usually determined as a percentage of the total bill
<ins>trapezoid</ins> a quadrilateral with a pair of parallel sides

Example:

<ins>triangle</ins> a polygon with three sides
<ins>value</ins> the worth of a number
<ins>volume</ins> the amount of space occupied by an object, expressed in cubic units where the formula is: **length x width x height**
<ins>whole number</ins> any number 0, 1, 2, 3, etc....

Mathematical Symbols

L = length
W = width
A = area
P = perimeter
s = side (of a square)
b = base
h = height of triangle or trapezoid [height can also be called altitude]
b_1 = parallel base (of trapezoid)
b_2 = parallel base (of trapezoid)
r = radius of a circle
D = diameter of a circle
C = circumference of a circle

π = pi = 3.14 = $\frac{22}{7}$

Metric Units

Temperature - Celsius

0°C	the freezing point of water
37°C	the normal body temperature
100°C	the boiling point of water

Mass

1000 milligrams (mg) = 1 gram
1000 grams = 1 kilogram (kg)
1000 kilograms = 1 metric ton (t)

Capacity

1000 milliliters (ml) = 1 liter (L)
1000 liters = 1 kiloliter (kL)

Length

10 millimeters (mm) = 1 centimeter
10 centimeters (cm) = 1 decimeter
1000 millimeters (mm) = 1 meter (m)
100 centimeters = 1 meter (m)
10 decimeters (dm) = 1 meter
1000 meters (m) = 1 kilometer (km)

Customary Units

Temperature - Fahrenheit

32°F	the freezing point of water
98.6°F	the normal body temperature
212°F	the boiling point of water

Weight

1 pound (LB) = 16 ounces (oz)
1 ton = 2,000 pounds

Time

1 minute (min) = 60 seconds (s)
1 hour = 60 minutes
1 day = 24 hours
1 week = 7 days
1 month (mo) = approximately 4 weeks

1 year (yr.)	=	365 days
		52 weeks
		12 months

1 decade = 10 years
1 century = 100 years

Capacity

1 cup (c)	=	8 fluid ounces (fl oz)
1 pint (pt)	=	16 fluid ounces
	=	2 cups
1 quart (qt)	=	32 fluid ounces
	=	4 cups
	=	2 pints
1 gallon(gal)	=	128 fluid ounces
	=	16 cups
	=	8 pints
	=	4 quarts

Length

1 foot (ft)	=	12 inches (in)
1 yard (yd)	=	36 inches
	=	3 feet
1 mile (mi)	=	5,280 feet
		1,760 yards

Conversions

1 inch (in)	=	2.54 cm
1 foot (ft)	=	30.48 cm
1 meter (m)	=	39.37 in
1 quart (qt)	=	1.07 liter
32°F	=	0°C
98.6°F	=	37°C
212°F	=	100°C
-40°F	=	-40°C
1 mile (mi)	=	1.609 km

Metric Conversion

left | **center** | *right*

millimeter	centimeter	decimeter	Meter	decameter	hectometer	kilometer
0.001	**0.01**	**0.1**	**1**	**10**	**100**	**1000**

To go from *left* to *right* requires division

from
millimeter to centimeter
divide by 10

from
centimeter to decimeter
divide by 10

from
decimeter to Meter
divide by 10

from
Meter to decameter
divide by 10

from
decameter to hectometer
divide by 10

from
hectometer to kilometer
divide by 10

left | **center** | *right*

millimeter	centimeter	decimeter	Meter	decameter	hectometer	kilometer
0.001	**0.01**	**0.1**	**1**	**10**	**100**	**1000**

Moving from *right* to *left*, requires multiplication

from
kilometer to hectometer
multiply by 10

from
hectometer to decameter
multiply by 10

from
decameter to Meter
multiply by 10

from
Meter to decimeter
multiply by 10

from
decimeter to centimeter
multiply by 10

from
centimeter to millimeter
multiply by 10

Conversion Techniques

In this conversion technique, always keep in mind the diagram from Page 6. The diagram illustates that going from the left of the table to the right requires division. For example, to convert 84 decimeters into kilometers would require that we divide the 84 decimeters by 10,000; the 10,000 conversion rate is the product of 10 four times, i.e., from decimeter to meter, from meter to decameter, from decameter to hectometer, and from hectometer to kilometer which is

$$10 \times 10 \times 10 \times 10 = 10^4 = 10{,}000$$

Therefore, to convert 84 decimeters into kilometers requires that:

$$84 \text{ decimeter} \div 10{,}000 = 0.0084 \text{ kilometer}$$

Another method would involve the technique commonly referred to as the Cancellation Technique. Start with the 84 decimeter and proceed as follows:

$$84 \text{ decimeter} \times \frac{1 \text{ meter}}{10 \text{ decimeter}} \times \frac{1 \text{ decameter}}{10 \text{ meters}} \times \frac{1 \text{ hectometer}}{10 \text{ decameters}} \times \frac{1 \text{ kilometer}}{10 \text{ hectometers}}$$

At each stage, you must cancel the numerator with a conversion (or equality) in the numerator. In this method the numerators and denominators cancel each other out so that only the desired unit is left.

$$84 \cancel{\text{ decimeter}} \times \frac{1 \cancel{\text{ meter}}}{10 \cancel{\text{ decimeter}}} \times \frac{1 \cancel{\text{ decameter}}}{10 \cancel{\text{ meters}}} \times \frac{1 \cancel{\text{ hectometer}}}{10 \cancel{\text{ decameters}}} \times \frac{1 \text{ kilometer}}{10 \cancel{\text{ hectometers}}}$$

In this example, all of the units have canceled except "kilometer" which is the desired unit.

Always keep in mind that the objective of this method is to have the numerators and denominators cancel each other out. In order to do this, you must use equalities which in effect cancel each other.

Prime Numbers

The integer 1 has just one divisor. Every other integer has at least two divisors because it is divisible by 1 and by itself. Integers with exactly two divisors are of great importance in number theory; they are called **primes**. [Number theory is the study of numbers and their properties.]

A prime is a positive integer greater than 1 that is divisible by no integers other than 1 and itself. The integers 2, 3, 5, 13, 101, and 163 are primes.

A positive integer which is not prime, and which is not equal to 1, is called **composite**. The integers 4 (2x2), 8 (4x2), 33 (3x11), 111 (3x37) and 1001 (7x11x13) are all composite. The primes are the building blocks of the integers. Every positive integer can be rewritten uniquely as the product of primes.

Every positive integer greater than 1 has a prime divisor.
There are infinitely many primes.
Every even positive integer greater than 2 can be written as the sum of two primes.

E.g., the integer 9 is a composite number because 9 has factors of 1, 3 and 9. 10 is a composite number because 10 has factors of 1, 2, 5 and 10. On the other hand, 11 is a prime number because it only has factors of 1 and itself.

In fact, all even numbers except 2 are composite.
Therefore, all even numbers greater than 2 are composite.
The number 1 is neither prime nor composite; it is a divisor.
Additionally, some odd numbers are composite, such as:
9, 15, 21, 25, 27, 33, 35, etc..

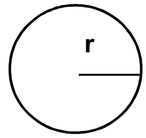
In summary,
The number 1 is neither prime nor composite because it is a universal divisor;
The only even prime number is 2;
The smallest prime number is 2; and
Therefore all prime numbers are odd except 2.

As an information, The set of prime numbers less than 100 are:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,
53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Area

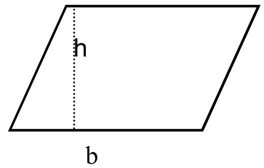
1 Area of circle = πr^2

where π = pi = 3.14 = $\frac{22}{7}$



2 Area of parallelogram = b x h

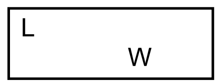
where b=base and h=height



> **Example**: if b=6 in and h=4 in, then
> Area = 6x4 = 24 sq. in = 24 in²

Note : the height of the parallelogram is always perpendicular to the base.

3 Area of rectangle = L x W

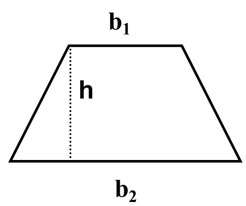


Example: if L=5 ft and W=7 ft, then Area = 5x7 = 35 sq. ft = 35 ft²

4 Area of square = s^2 = s x s

> **Example**: if s=6 inches,
> then Area = 6 x 6 = 6² = 36 sq. inches = 36 in²

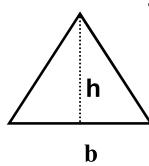
5 Area of trapezoid = $\frac{h}{2}$ x (b_1 + b_2)



> **Example**: if b_1=7cm, and b_2=13cm,
> and h=5cm, Area = 5/2 x (7+13) =
> 5 x 10 = 50 sq. cm = 50 cm²

Note : the bases of the trapezoid are always parallel to each other.

6 Area of triangle = ½ x (bxh) = $\frac{b \text{ x } h}{2}$



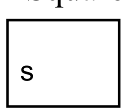
> **Example**: if b=11 inches and
> h= 13 inches, then Area = 1/2 x (11x13)
> = 1/2 x (143) = 71.5 sq. in = 71.5 in²

Note : the height of the triangle is always perpendicular to the base.

Perimeter

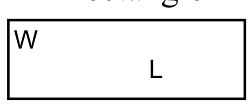
Perimeter of quadrilateral:

Square



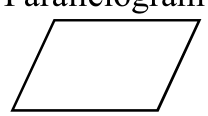
if s = 6cm,
then, P = 4s
P=4x6=24cm

Rectangle



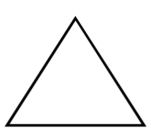
if L=10cm and W=3cm
then, P = 2L + 2W
P = 2(L + W)
P = 2(10+3)=26cm

Parallelogram



if adjacent sides are
8cm and 4cm, then
P = sum of the 4 sides
P = 4+8+4+8=24cm

Perimeter of triangle:



if the sides are 5cm, 7cm, and 9cm, then
P = sum of the 3 sides
P = 5 + 7 + 9 = 21cm

Multiplication of Numerals with signs

A. **positive x positive = positive; (+)(+) = +**

Example: **(+6)(+8) = 48**

Example: **(4)(9) = 36**

Note: A positive integer does not need to show its sign

B. **negative x negative = positive; (-)(-) = +**

Example: **(-5)(-7) = 35**

C. **positive x negative = negative; (+)(-) = -**

Example: **(+3)(-2) = (3)(-2) = -6**

D. **negative x positive = negative; (-)(+) = -**

Example: **(-17)(+4) = (-17)(4) = -68**

Multiplication of Variables

Multiplication of variables involves algebraic terms.

An algebraic term is composed of a coefficient and a variable.

A coefficient is a number whereas,

A variable is an unknown number represented by an alphabet.

The steps involved are:
1) multiply coefficients by coefficients, and
2) multiply variables by variables

Example: $(7xyz^3)(-4xy^2z) = -28x^2y^3z^4$

$= 7 \times -4 = -28;\ (x)(x) = x^2;\ (y)(y^2) = y^3;\ (z^3)(z) = z^4$

Least Common Multiple [LCM]

The least Common Multiple and the Least Common Denominator are one in the same.

1) If the integers (or denominators) are a factor of each other, then select the larger integer; for example, if provided 5 and 10, then select 10.
2) If the integers (or denominators) have nothing in common, then the LCM is the product of the integers; for example, if provided 5 and 17, then select 85, which is 5 x 17.
3) If the integers (or denominators) have something in common and are not factors of each other, then separate the integers into prime numbers and exponents. The LCM is the product of the largest exponents of the prime numbers, **for example**, 18 and 24 would be:

$$18 = 2 \times 3^2 = 2 \times 9$$
$$24 = 2^3 \times 3 = 8 \times 3$$

The LCM would be:

$$72 = 2^3 \times 3^2 = 8 \times 9$$

Greatest Common Factor [GCF]

1) Separate the integers into all common terms
2) Determine the greatest (largest) of all of the common terms
   If the integers are factors of each other then the GCF is the lesser of the integers
   for example, 72 and 144 has a GCF of 72
3) Be careful! You may have to multiply common terms together
   for example, 150 and 210,

$$150 = 2 \times 3 \times 5^2 = 2 \times 3 \times 25$$
$$210 = 2 \times 3 \times 5 \times 7 = 2 \times 3 \times 5 \times 7$$

   then the GCF is: $30 = 2 \times 3 \times 5$

4) Keep in mind that the GCF presumes that the integers have common factors
5) Lastly, the GCF is basically the opposite of the LCM in that the GCF uses
   the common terms in the **least** number of times whereas the
   the LCM uses the common terms in the **most** number of times

Percent

$$\text{Percent} = \frac{\text{part}}{\text{whole}} = \frac{\text{part}}{\text{total}} = \frac{\text{per}}{100}$$

Example: if there are 5 black, 3 red and 2 blue marbles,
then the percent of Black marbles is:

$$\frac{\text{part}}{\text{total}} = \frac{5}{5+3+2} = \frac{5}{10} = \frac{1}{2} = 50\%$$

number of items or objects = pct% x [total]

Example: if the total population is 10,000 and the percentage of boys is 30%,
then the number of boys = 30% x 10,000 = 30 x 10,000 = 0.3 x 10,000 = 3,000

total cost after sales tax = cost + (cost x sales tax%) = cost (1+sales tax%)

Example: if the cost of the book is $15, and the sales tax is 8% then the total cost is:
= $15 + (8% of $15) = $15 + $1.20 = $16.20 **Or**
= $15(1 + 8%) = $15(1+0.08) = $15(1.08) = $16.20

Factorial

n! [is read n factorial] = (n)(n-1)(n-2)(n-3) ... (1)
where ! is the factorial function

Example:

3! = (3)(2)(1) = 3 x 2 x 1 = 6

4! = (4)(3)(2)(1) = 4 x 3 x 2 x 1 = 24

6! = (6)(5)(4)(3)(2)(1) = 6 x 5 x 4 x 3 x 2 x 1 = 720

8! = (8)(7)(6)(5)(4)(3)(2)(1) = 8 x 7x 6 x 5 x 4 x 3 x 2 x 1 = 40,320

However:

4! = (4)(3!) = 4 x 3! = 4 x 6 = 24

5! = (5)(4!) = 5 x 4! = 5 x 24 = 120

9! = (9)(8!) = 9 x 8! = 9 x 40,320 = 362,880

n! = (n)(n-1!) = n x (n-1)!

Scientific Notation

6.23×10^{35} = scientific notation = means that the number is to be rewritten in the form of: **$A.bc \times 10^{n}$** where A is between 1 and 10

Example: 13,686,441,128 = 1.37×10^{10}

Ratio

Q. What is the ratio of boys to girls in Rodney's class if there are 13 boys and 7 girls

A. 13: 7 = $\frac{13}{7}$ $\frac{\text{boys}}{\text{girls}}$ = 13 boys: 7 girls

Q. What is the percent of boys in Rodney's class

A. $\frac{\text{part}}{\text{total}}$ $\frac{13}{20} = \frac{65}{100} = 65\%$

Q. What is the ratio of peanuts consumption to crackers consumption if 60% of the population eats peanuts and 40% eats crackers

A. 60 : 40 = $\frac{60}{40} = \frac{3}{2}$

A rate is a ratio that compares two different units of measure.

Example: Kemet entered 40 pages of data into the computer in 5 hours. What is the rate per hour?

$\frac{\text{40 pages}}{\text{5 hours}}$ Compare the number of pages to the number of hours.

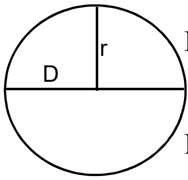
$\frac{\text{8 pages}}{\text{1 hour}}$ Simplify to find the **unit rate**, which is the number of pages entered per hour.

A proportion is an equation that shows that two ratios are equivalent. To determine if a pair of ratios form a proportion, you can find their cross products. If the cross products are equal, the ratios form a proportion.

Q. Do $\frac{5}{8}$ and $\frac{17.5}{28}$ form a proportion?

A. Yes Cross multiply: 5×28 and 8×17.5; The two ratios are equivalent because $5 \times 28 = 140$ and $8 \times 17.5 = 140$, therefore they are equal

Circle



Radius = r = distance from center of Circle to any point on the circle.

Diameter = D = chord that passes through the center of the circle from one side to another side

Diameter = 2r = 2 x the radius = twice the radius

Example: if r = 8 cm, then D = 2x8 = 16 cm

Area = $\pi \mathbf{r}^2$ where π = 3.14

Example: if r = 7 cm, then Area = $\pi r^2 = 3.14 \times (7^2) = 3.14 \times 49 = 49\pi$ sq. cm

Or = 153.86 sq. cm = 153.86 cm^2 = 49π cm^2

Circumference = $2\pi r$ = 2 x π x r = Dπ = D x π

Example:, if r = 9 cm, then Circumference = 2 x π x 9 = 2 x 3.14 x 9 = 56.52 cm

Or Circumference = 18π

Remember, the circumference is the distance around the circle.
In other words, the circumference is equivalent to calculating the perimeter of the circle.

Exponents

An exponent is a number or symbol denoting the power to which another number or expression is to be raised.

Example: 5 x 5 = 25 however, we can rewrite 5 x 5 by 5^2
where 5 is the **base** and 2 is the **exponent**.

Example: 6 x 6 x 6 = 6^3 Note that 6 x 6 x 6 = 6^3 = 216
This is because 6 is multiplied by itself three times.

Example: p x p x p = p^3 This is because p is multiplied by itself three times.

Example: 7 x 7 x 7 x 7 x 7 = 7^5 Note that 7 x 7 x 7 x 7 x 7 = 16,807
This is because 7 is multiplied by itself five times, that is, 7 is the base, while 5 is the exponent.

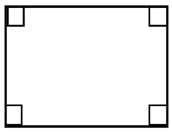
Example: 8 x 8 = 8^2 = 64 Note that 8 x 8 = 64
This is because 8 is multiplied by itself twice.
The exponent is 2, that is, 8 is raised to the second power.

Whenever the exponent is 2, we call it "squared". Therefore, 7^2 is read as 7 squared.
When the exponent is 3, we call it "cubed". Subsequently, 9^3 is read as 9 cubed.
Lastly, any base raised to the zero exponent, such as 5^0 is equal to 1.
In other words, any base raised to the zero power is equal to 1. $\mathbf{x^0 = 1}$

Geometrical Figures

A Quadrilateral is a four-sided polygon.
Quadrilaterals may be classified by looking at their sides and angles.

Square

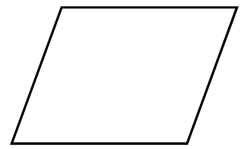


All sides are congruent. All four angles are right angles.

Rectangle

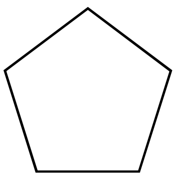
Both pairs of opposite sides are parallel. All four angles are right angles.

Parallelogram

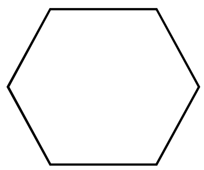


Both pairs of opposite sides are parallel.

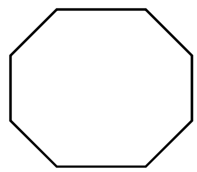
A polygon is named according to the number of its sides.



pentagon
[5 sides]



hexagon
[6 sides]

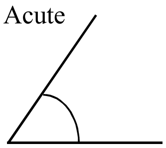


octagon
[8 sides]

Angles

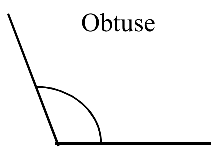
An angle is the measured space between two lines that meet at one point.
There are three types of angles: Acute, Obtuse and Right.

Acute



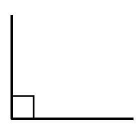
An angle measuring less than 90°

Obtuse



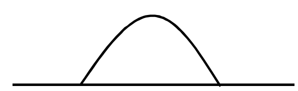
An angle measuring more than 90° and less than 180°

Right



An angle measuring exactly 90°; therefore it is perpendicular

Straight



An angle measuring exactly 180°

The sum of the three angles in a triangle must measure **180°**,
and the sum of the four angles in a quadrilateral must measure **360°**.

Divisibility

To determine whether an integer is divisible by:

2	the integer must be an even integer **Example**: 2; 4; 6; 8; and 120
3	the sum of the digits must add up to a multiple of 3 **Example**: 3; 123 (adds up to 6); and 815,271 (adds up to 24 which adds up to 6)
4	the integer must be an even integer and can be divided by 2 twice Also, the integer's last two digits are a factor of 4 **Example**: 78 is even and is **not** divisible by 4 while 176 is even and is divisible by 2 twice, that is 176 = 44 x 2 x 2 = 44 x 4; therefore it is **divisible** by 4; Also, 76, which is the last two digits, is divisible by 4
5	the integer ends in 5 or 0 **Example**: 25; 625,690; and 7,895
6	the integer must be an even integer and divisible by 3, that is, it is divisible by 2 **and** divisible by 3 (÷ 2) and (÷ 3) = (÷ 6) Or (÷ 2) x (÷ 3) = ÷ 6 **Example**: 33,120 is even and is divisible by 3 so therefore it is divisible by 6 however 33,111 is divisible by 3 and is **not** even, so therefore it is **not** divisible by 6
8	the integer must be an even integer and can be divided by 2 three times, hence $2^3 = 8$; that is, 2 x 2 x 2 = 8 **Example**: 28 is even and is **not** divisible by 8 whereas 56 is even and is divisible by 8 because 56 = 7 x 2 x 2 x 2 = 7 x 8 = 56
9	the sum of the digits must add up to a multiple of 9 **Example**: 780 is divisible by 3, but it is **not** divisible by 9 while 441 is divisible by 9 (it is also divisible by 3)
10	the integer must end in zero **Example**: 2,140; 56,000; and 1,230
11	the sum of the alternating digits must cancel out **Example**: 7,502 is divisible by 11 where the 1st and 3rd digits from the right add up to 7, (2+5) and where the 2nd and 4th digits from the right add up to 7, (0+7) so therefore, the sums of the digits cancel out, 7 = 7, which means 7,502 is divisible by 11 **Example**: 121 is divisible by 11 where the 1st and 3rd digits add up to 2, (1+1) and where the 2nd digit adds up to 2, so therefore, the sums of the digits cancel out, 2 = 2, **ergo** 121 is divisible by 11

Squares & Square Roots

Squares of 1 through 30

1 x 1	=	1^2	=	1
2 x 2	=	2^2	=	4
3 x 3	=	3^2	=	9
4 x 4	=	4^2	=	16
5 x 5	=	5^2	=	25
6 x 6	=	6^2	=	36
7 x 7	=	7^2	=	49
8 x 8	=	8^2	=	64
9 x 9	=	9^2	=	81
10 x 10	=	10^2	=	100
11 x 11	=	11^2	=	121
12 x 12	=	12^2	=	144
13 x 13	=	13^2	=	169
14 x 14	=	14^2	=	196
15 x 15	=	15^2	=	225
16 x 16	=	16^2	=	256
17 x 17	=	17^2	=	289
18 x 18	=	18^2	=	324
19 x 19	=	19^2	=	361
20 x 20	=	20^2	=	400
21 x 21	=	21^2	=	441
22 x 22	=	22^2	=	484
23 x 23	=	23^2	=	529
24 x 24	=	24^2	=	576
25 x 25	=	25^2	=	625
26 x 26	=	26^2	=	676
27 x 27	=	27^2	=	729
28 x 28	=	28^2	=	784
29 x 29	=	29^2	=	841
30 x 30	=	30^2	=	900

Square Root of 1 through 30

$\sqrt{1}$	=	1.00
$\sqrt{2}$	=	1.41
$\sqrt{3}$	=	1.73
$\sqrt{4}$	=	2.00
$\sqrt{5}$	=	2.24
$\sqrt{6}$	=	2.45
$\sqrt{7}$	=	2.65
$\sqrt{8}$	=	2.83
$\sqrt{9}$	=	3.00
$\sqrt{10}$	=	3.16
$\sqrt{11}$	=	3.32
$\sqrt{12}$	=	3.46
$\sqrt{13}$	=	3.61
$\sqrt{14}$	=	3.74
$\sqrt{15}$	=	3.87
$\sqrt{16}$	=	4.00
$\sqrt{17}$	=	4.12
$\sqrt{18}$	=	4.24
$\sqrt{19}$	=	4.36
$\sqrt{20}$	=	4.47
$\sqrt{21}$	=	4.58
$\sqrt{22}$	=	4.69
$\sqrt{23}$	=	4.80
$\sqrt{24}$	=	4.90
$\sqrt{25}$	=	5.00
$\sqrt{26}$	=	5.10
$\sqrt{27}$	=	5.20
$\sqrt{28}$	=	5.29
$\sqrt{29}$	=	5.39
$\sqrt{30}$	=	5.48

Perfect Square Roots

$\sqrt{1}$	=	1
$\sqrt{4}$	=	2
$\sqrt{9}$	=	3
$\sqrt{16}$	=	4
$\sqrt{25}$	=	5
$\sqrt{36}$	=	6
$\sqrt{49}$	=	7
$\sqrt{64}$	=	8
$\sqrt{81}$	=	9
$\sqrt{100}$	=	10
$\sqrt{121}$	=	11
$\sqrt{144}$	=	12
$\sqrt{169}$	=	13
$\sqrt{196}$	=	14
$\sqrt{225}$	=	15
$\sqrt{256}$	=	16
$\sqrt{289}$	=	17
$\sqrt{324}$	=	18
$\sqrt{361}$	=	19
$\sqrt{400}$	=	20
$\sqrt{441}$	=	21
$\sqrt{484}$	=	22
$\sqrt{529}$	=	23
$\sqrt{576}$	=	24
$\sqrt{625}$	=	25
$\sqrt{676}$	=	26
$\sqrt{729}$	=	27
$\sqrt{784}$	=	28
$\sqrt{841}$	=	29
$\sqrt{900}$	=	30

Median

To calculate the median, you must:

1) Order the list of items from least to greatest
2) The median is the middle term
3) If there are an **odd** number of items then the median = ½ of [n+1] items, for example if there are n=7 items, then the median is the 4th ordered item because ½ of (7+1) = ½ of 8 = 4
4) If there are an **even** number of items then the median = midpoint of the middle items, **for example**, if there are n=16 items, then the median is the midpoint of the 8th and 9th ordered items

n=odd, n=7	n=even, n=10
15	29
17	37
18	39
23	41
36	**46**
49	**50**
59	58
	59
	62
	69

The **median = 23**, the 4th term in order of least to greatest

The **median = 48**, the midpoint of the 5th and 6th term, that is the Midpoint of 46 and 50, which is 48

Mean

The mean is simply a number obtained by adding a group of numbers together and dividing by the number of addends. Another name for the **mean** is the **average**.

for example,

	89
	78
	100
	63
+	50
Sum	380

Note: Mean = Average

$$= \frac{\text{sum of items}}{\text{number of items}} = \frac{\text{sum of addends}}{\text{number of addends}}$$

Number of addends = 5 therefore,

$$\text{Mean} = \text{Average} = 380 \quad \frac{380}{5} = 76$$

Fractions & Decimals

$\frac{1}{2}$	=	0.5	$\frac{1}{6}$	=	0.167	$\frac{1}{8}$	=	0.125
$\frac{1}{3}$	=	0.333	$\frac{5}{6}$	=	0.833	$\frac{3}{8}$	=	0.375
$\frac{2}{3}$	=	0.667	$\frac{1}{7}$	=	0.143	$\frac{5}{8}$	=	0.625
$\frac{1}{4}$	=	0.25	$\frac{2}{7}$	=	0.286	$\frac{7}{8}$	=	0.875
$\frac{3}{4}$	=	0.75	$\frac{3}{7}$	=	0.429	$\frac{1}{9}$	=	0.111
$\frac{1}{5}$	=	0.2	$\frac{4}{7}$	=	0.571	$\frac{2}{9}$	=	0.222
$\frac{2}{5}$	=	0.4	$\frac{5}{7}$	=	0.714	$\frac{4}{9}$	=	0.444
$\frac{3}{5}$	=	0.6	$\frac{6}{7}$	=	0.857	$\frac{5}{9}$	=	0.556
$\frac{4}{5}$	=	0.8	$\frac{1}{10}$	=	0.1	$\frac{7}{9}$	=	0.778
$\frac{1}{11}$	=	0.091	$\frac{1}{12}$	=	0.083	$\frac{8}{9}$	=	0.889
$\frac{5}{12}$	=	0.417	$\frac{7}{12}$	=	0.583	$\frac{1}{15}$	=	0.067
$\frac{1}{16}$	=	0.063	$\frac{1}{25}$	=	0.04	$\frac{1}{32}$	=	0.031

Base 10 and Other Numeral Systems

Everything we do in terms of numbers is in the decimal system. In other words, they are in the base 10 system. Many times we are asked to convert into another base. For example, we may be asked to convert base 4 into base 10.
First, let us review exactly what is meant by base 10.

123,456 =	100,000	=	1×10^5	=	1 x 100,000
	20,000	=	2×10^4	=	2 x 10,000
	3,000	=	3×10^3	=	3 x 1,000
	400	=	4×10^2	=	4 x 100
	50	=	5×10^1	=	5 x 10
	+ 6	=	6×10^0	=	6 x 1
	123,456		123,456		123,456

Now we are ready to convert $32012_{\mathbf{base4}}$ to an equivalent $_{\mathbf{base10}}$ number.

	base4		**base10**
$32012_{\mathbf{base4}}$ =	3×4^4	=	768
	2×4^3	=	128
	0×4^2	=	0
	1×4^1	=	4
	+ 2×4^0	=	2
	$32012_{\mathbf{base4}}$		902

Therefore,
$32012_{\mathbf{base4}} = 902_{\mathbf{base10}} = 902$

Keep in mind that in any base numeral system that the largest single integer found in the numeral system is **one less** than the base numeral system.
For example, in the **base10** numeral system, the largest single integer is 9.
Therefore, in the **base4** numeral system, the largest single integer is 3.
As can be expected, in the **base2** numeral system, the largest single integer is 1.

The **base2** numeral system is very important; it is called the **binary** numeral system, and is populated by just two integers: **0 and 1** which are the cornerstone of artificial intelligence and computer logic.
It should be obvious that **base2** numeral system is the smallest numeral system.

	base2		**base10**
$1101_{\mathbf{base2}}$ =	1×2^3	=	8
	1×2^2	=	4
	0×2^1	=	0
	+ 1×2^0	=	1
	$1101_{\mathbf{base2}}$ =		13

Therefore,
$1101_{\mathbf{base2}} = 13_{\mathbf{base10}} = 13$

Multiplication Tables

1x	2x	3x	4x	5x	6x
1 x 1 = 1	2 x 1 = 2	3 x 1 = 3	4 x 1 = 4	5 x 1 = 5	6 x 1 = 6
1 x 2 = 2	2 x 2 = 4	3 x 2 = 6	4 x 2 = 8	5 x 2 = 10	6 x 2 = 12
1 x 3 = 3	2 x 3 = 6	3 x 3 = 9	4 x 3 = 12	5 x 3 = 15	6 x 3 = 18
1 x 4 = 4	2 x 4 = 8	3 x 4 = 12	4 x 4 = 16	5 x 4 = 20	6 x 4 = 24
1 x 5 = 5	2 x 5 = 10	3 x 5 = 15	4 x 5 = 20	5 x 5 = 25	6 x 5 = 30
1 x 6 = 6	2 x 6 = 12	3 x 6 = 18	4 x 6 = 24	5 x 6 = 30	6 x 6 = 36
1 x 7 = 7	2 x 7 = 14	3 x 7 = 21	4 x 7 = 28	5 x 7 = 35	6 x 7 = 42
1 x 8 = 8	2 x 8 = 16	3 x 8 = 24	4 x 8 = 32	5 x 8 = 40	6 x 8 = 48
1 x 9 = 9	2 x 9 = 18	3 x 9 = 27	4 x 9 = 36	5 x 9 = 45	6 x 9 = 54
1 x 10 = 10	2 x 10 = 20	3 x 10 = 30	4 x 10 = 40	5 x 10 = 50	6 x 10 = 60
1 x 11 = 11	2 x 11 = 22	3 x 11 = 33	4 x 11 = 44	5 x 11 = 55	6 x 11 = 66
1 x 12 = 12	2 x 12 = 24	3 x 12 = 36	4 x 12 = 48	5 x 12 = 60	6 x 12 = 72
1 x 13 = 13	2 x 13 = 26	3 x 13 = 39	4 x 13 = 52	5 x 13 = 65	6 x 13 = 78
1 x 14 = 14	2 x 14 = 28	3 x 14 = 42	4 x 14 = 56	5 x 14 = 70	6 x 14 = 84
1 x 15 = 15	2 x 15 = 30	3 x 15 = 45	4 x 15 = 60	5 x 15 = 75	6 x 15 = 90
1 x 16 = 16	2 x 16 = 32	3 x 16 = 48	4 x 16 = 64	5 x 16 = 80	6 x 16 = 96

7x	8x	9x	10x	11x	12x
7 x 1 = 7	8 x 1 = 8	9 x 1 = 9	10 x 1 = 10	11 x 1 = 11	12 x 1 = 12
7 x 2 = 14	8 x 2 = 16	9 x 2 = 18	10 x 2 = 20	11 x 2 = 22	12 x 2 = 24
7 x 3 = 21	8 x 3 = 24	9 x 3 = 27	10 x 3 = 30	11 x 3 = 33	12 x 3 = 36
7 x 4 = 28	8 x 4 = 32	9 x 4 = 36	10 x 4 = 40	11 x 4 = 44	12 x 4 = 48
7 x 5 = 35	8 x 5 = 40	9 x 5 = 45	10 x 5 = 50	11 x 5 = 55	12 x 5 = 60
7 x 6 = 42	8 x 6 = 48	9 x 6 = 54	10 x 6 = 60	11 x 6 = 66	12 x 6 = 72
7 x 7 = 49	8 x 7 = 56	9 x 7 = 63	10 x 7 = 70	11 x 7 = 77	12 x 7 = 84
7 x 8 = 56	8 x 8 = 64	9 x 8 = 72	10 x 8 = 80	11 x 8 = 88	12 x 8 = 96
7 x 9 = 63	8 x 9 = 72	9 x 9 = 81	10 x 9 = 90	11 x 9 = 99	12 x 9 = 108
7 x 10 = 70	8 x 10 = 80	9 x 10 = 90	10 x 10 = 100	11 x 10 = 110	12 x 10 = 120
7 x 11 = 77	8 x 11 = 88	9 x 11 = 99	10 x 11 = 110	11 x 11 = 121	12 x 11 = 132
7 x 12 = 84	8 x 12 = 96	9 x 12 = 108	10 x 12 = 120	11 x 12 = 132	12 x 12 = 144
7 x 13 = 91	8 x 13 = 104	9 x 13 = 117	10 x 13 = 130	11 x 13 = 143	12 x 13 = 156
7 x 14 = 98	8 x 14 = 112	9 x 14 = 126	10 x 14 = 140	11 x 14 = 154	12 x 14 = 168
7 x 15 = 105	8 x 15 = 120	9 x 15 = 135	10 x 15 = 150	11 x 15 = 165	12 x 15 = 180
7 x 16 = 112	8 x 16 = 128	9 x 16 = 144	10 x 16 = 160	11 x 16 = 176	12 x 16 = 192

13x	14x	15x	16x
13 x 1 = 13	14 x 1 = 14	15 x 1 = 15	16 x 1 = 16
13 x 2 = 26	14 x 2 = 28	15 x 2 = 30	16 x 2 = 32
13 x 3 = 39	14 x 3 = 42	15 x 3 = 45	16 x 3 = 48
13 x 4 = 52	14 x 4 = 56	15 x 4 = 60	16 x 4 = 64
13 x 5 = 65	14 x 5 = 70	15 x 5 = 75	16 x 5 = 80
13 x 6 = 78	14 x 6 = 84	15 x 6 = 90	16 x 6 = 96
13 x 7 = 91	14 x 7 = 98	15 x 7 = 105	16 x 7 = 112
13 x 8 = 104	14 x 8 = 112	15 x 8 = 120	16 x 8 = 128
13 x 9 = 117	14 x 9 = 126	15 x 9 = 135	16 x 9 = 144
13 x 10 = 130	14 x 10 = 140	15 x 10 = 150	16 x 10 = 160
13 x 11 = 143	14 x 11 = 154	15 x 11 = 165	16 x 11 = 176
13 x 12 = 156	14 x 12 = 168	15 x 12 = 180	16 x 12 = 192
13 x 13 = 169	14 x 13 = 182	15 x 13 = 195	16 x 13 = 208
13 x 14 = 182	14 x 14 = 196	15 x 14 = 210	16 x 14 = 224
13 x 15 = 195	14 x 15 = 210	15 x 15 = 225	16 x 15 = 240
13 x 16 = 208	14 x 16 = 224	15 x 16 = 240	16 x 16 = 256

Sequences: Arithmetic

A sequence is a series of numbers coming one after another in a certain order. A sequence can be either **finite** - where it has a termination point or **infinite** - where it goes on forever. There are two types of sequences: **arithmetic** and **geometric**. An arithmetic sequence is a sequence where the individual numbers in the sequence differ by the same added or subtracted number, that is to say that the same number is being added or subtracted to each term. **Example: 4,7,10,13,16,…** where each successive term is 3 more than the prior term

A geometric sequence is a sequence where the individual numbers in the sequence differ by the same multiplied or divided by term, that is to say that the same number is being multiplied or divided into each term. **Example: ¼, ½, 1, 2, 4, 8, 16, ...** where each successive term is twice as much as the prior term

Example 20.1 : 1, 3, 5, 7, 9 … is an arithmetic sequence of the odd numbers (where each term differs by 2)

let a_1 = the **first term** in the sequence

let n = the **number of terms** in the sequence

let d = the **difference** between each successive term in the sequence

then S_n is the **sum** of the first n terms in the sequence

and a_n = the **nth term** of the sequence; for example the 5th term is called a_5

Therefore:

formula 20.1 ⟶ $a_n = a_1+(n-1)d$

formula 20.2 ⟶ $S_n = \frac{n(a_1 + a_n)}{2}$

formula 20.3 ⟶ $S_n = \frac{n[2a_1+(n-1)d]}{2}$

Please note that the difference between formula 20.2 and formula 20.3 is that 20.3 does not require the knowledge of the last term in order to calculate the sum of the sequence.

In the Example 20.1, the first term (a_1) is 1, and the difference (d) between each consecutive term is 2, thus:

⟶ **$a_1 = 1$; $d = 2$; $a_7 = 13$ because $a_7 = a_1+(7-1)x2 = 1 + (7-1)x2 = 1 + 12 = 13$**

that is, the 7th term in the arithmetic sequence of odd numbers is 13, which we know to be true.

⟶ $S_7 = 49$; the sum of the first 7 terms, S_7,

$$S_7 = \frac{n(a_1+a_n)}{2} = \frac{7(1+13)}{2} = 49$$

Summation

$\sum$ is the symbol for the summation of a sequence; it is the Greek letter - sigma - meaning Summation or Sum.

formula 21.1

$$\sum_{i}^{n} i = \frac{(n)(n+1)}{2}$$

This is read as the sum of the sequence from i to n is n times (n plus 1) all divided by 2

Example: $\sum_{i=1}^{n=7} i$

The example is the sum of the sequence from 1 to 7:
Let, n=7, i=1

$$= \frac{(7)(7+1)}{2} = \frac{56}{2} = 28$$

This is true because 1+2+3+4+5+6+7 = 28

formula 21.2

$$\sum_{i}^{n} i^2 = \frac{(n)(n+1)(2n+1)}{6}$$

This is the sum of the squares from i to n is n times (n plus 1) times 2n plus 1 all divided by 6

Example: $\sum_{i=1}^{n=4} i^2$

The example is the sum of the sequence from 1 squared to 4 squared or the sum of the first 4 squares
Let, n=7, i=1

$$= \frac{(4)(4+1)(2x4+1)}{6} = \frac{(4)(5)(9)}{6}$$

$$= 30$$

This is true because $1^2+2^2+3^2+4^2$ = 1+4+9+16 = 30

formula 21.3

$$\sum_{i}^{n} i^3 = \left[\frac{(n)(n+1)}{2}\right]^2 = \left[\sum_{i}^{n} i\right]^2$$

This is left as an exercise for the student.

Geometric Sequences

As introduced on Page 20, a Sequence is a series of numbers coming one after another in a certain order. There are two types of sequences: **arithmetic** and **geometric**.

A geometric sequence is a sequence where the individual numbers in the sequence differ by the same multiplied or divided by term, that is to say that the same number is being multiplied or divided into each term.

Example 22.1: ¼, ½, 1, 2, 4, 8, 16, 32
Example 22.2: 1, 3, 9, 27, 81, 324
Example 22.3: 2, 1/2, 1/8, 1/32

In Example 22.1, the common term (known as the multiplicative ratio) is 2; Notice that each successive term is twice as much as the prior term. In a geometric series the common term is multiplicative of the other terms. When we say multiplicative, we mean that it can be multiplied or divided evenly into the other terms.
What are the multiplicative ratios for **Examples 22.2** and **22.3**?

Let a_1 be the first term of the geometric sequence

Let n be the nth term of the geometric sequence

Let r be the ratio (or the multiplicative difference) of any two consecutive terms of the geometric sequence

Then, the nth term of the geometric sequence is calculated by:

$$a_n = a_1 r^{(n-1)}$$ **formula 22.1**

Exercise: Based on **Example 22.1**, $a_3 = ¼ \times 2^{3-1} = ¼ \times 2^2 = ¼ \times 4 = 1$, which we know to be true because $r = 2$ and $a_3 = 1$

And the sum of the first n terms is:

$$S_n = \left[\frac{1 - r^n}{1 - r}\right] a_1$$ **formula 22.2**

Exercise: Based on **Example 22.2**, $s_4 = 1 \times \frac{(1-3^4)}{(1-3)} = \frac{-80}{-2} = 40$

This is true because 1 + 3 + 9 + 27 = 40, the sum of the first 4 terms in Example 22.2.

There is also a special type of geometric sequence that decreases forever. They are called **Infinitely Decreasing Series** where the sum is calculated by:

$$S = \frac{a_1}{1 - r}$$ **formula 22.3**